



Cambridge International AS & A Level

CANDIDATE
NAME

CENTRE
NUMBER

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MATHEMATICS

9709/33

Paper 3 Pure Mathematics 3

October/November 2023

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

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1 Find the set of values of x satisfying the inequality $|2^{x+1} - 2| < 0.5$, giving your answer to 3 significant figures. [4]

2 On an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z - 1 + 2i| \leq |z|$ and $|z - 2| \leq 1$. [5]

3 The polynomial $2x^3 + ax^2 + bx + 6$, where a and b are constants, is denoted by $p(x)$. When $p(x)$ is divided by $(x + 2)$ the remainder is -38 and when $p(x)$ is divided by $(2x - 1)$ the remainder is $\frac{19}{2}$.

Find the values of a and b .

[5]

4 Solve the quadratic equation $(3 + i)w^2 - 2w + 3 - i = 0$, giving your answers in the form $x + iy$, where x and y are real. [5]

5 Find the exact coordinates of the stationary points of the curve $y = \frac{e^{3x^2-1}}{1-x^2}$. [6]

6 (a) Show that the equation $\cot^2 \theta + 2 \cos 2\theta = 4$ can be written in the form

$$4 \sin^4 \theta + 3 \sin^2 \theta - 1 = 0.$$

[3]

(b) Hence solve the equation $\cot^2 \theta + 2 \cos 2\theta = 4$, for $0^\circ < \theta < 360^\circ$. [3]

7 The equation of a curve is $x^3 + y^2 + 3x^2 + 3y = 4$.

(a) Show that $\frac{dy}{dx} = -\frac{3x^2 + 6x}{2y + 3}$. [3]

(b) Hence find the coordinates of the points on the curve at which the tangent is parallel to the x -axis. [5]

8 The variables x and y satisfy the differential equation

$$e^{4x} \frac{dy}{dx} = \cos^2 3y.$$

It is given that $y = 0$ when $x = 2$.

Solve the differential equation, obtaining an expression for y in terms of x .

[7]

9 Let $f(x) = \frac{17x^2 - 7x + 16}{(2 + 3x^2)(2 - x)}$.

(a) Express $f(x)$ in partial fractions.

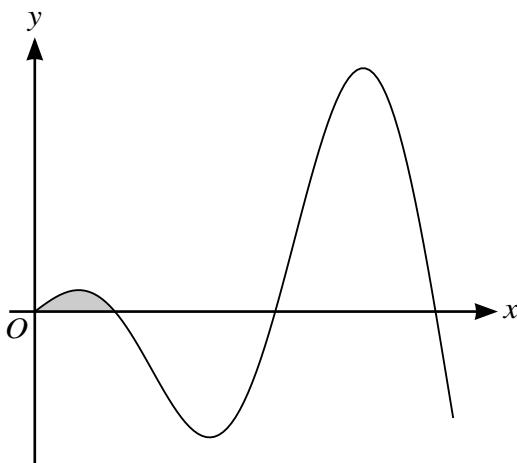
[5]

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^3 . [5]

(c) State the set of values of x for which the expansion in (b) is valid. Give your answer in an exact form. [1]

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10



The diagram shows the curve $y = x \cos 2x$, for $x \geq 0$.

(a) Find the equation of the tangent to the curve at the point where $x = \frac{1}{2}\pi$. [4]

(b) Find the exact area of the shaded region shown in the diagram, bounded by the curve and the x -axis. [5]

11 The line l has equation $\mathbf{r} = \mathbf{i} - 2\mathbf{j} - 3\mathbf{k} + \lambda(-\mathbf{i} + \mathbf{j} + 2\mathbf{k})$. The points A and B have position vectors $-2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $3\mathbf{i} - \mathbf{j} + \mathbf{k}$ respectively.

(a) Find a unit vector in the direction of l .

[2]

The line m passes through the points A and B .

(b) Find a vector equation for m .

[2]

(c) Determine whether lines l and m are parallel, intersect or are skew. [5]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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